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International Journal of Heat and Mass Transfer 50 (2007) 1637–1640
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Technical Note

# Laminar flow and heat transfer in plate-fin triangular ducts in thermally developing entry region

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Received 7 June 2006; received in revised form 22 September 2006
Available online 13 November 2006

**Abstract**

Laminar forced flow and heat transfer in plate-fin isosceles triangular ducts encountered in compact heat exchangers is investigated. The flow is hydrodynamically fully developed, but developing thermally under uniform temperature conditions. Heat conduction in the fin of finite conductance and convection in the fluid are analyzed simultaneously as a conjugate problem. The study covers a wide range of apex angles from 30° to 120°, and fin conductance parameters from 0 to infinitely large. Nusselt numbers in the developing and fully developed regions for various apex angles and fin conductance parameters are obtained, which can be used in estimation of heat transfer characteristics in plate-fin compact heat exchangers with fins of various conductivities and thickness.


*Keywords:* Plate-fin ducts; Triangular; Heat transfer; Compact heat exchanger

## 1. Introduction

In practice, it is necessary in many situations to use a heat exchanger with a very high ratio of heat transfer area to overall volume. A compact heat exchanger with triangular cross-sectional ducts, as shown in Fig. 1, is usually a good choice due to its excellent compactness and cost-effectiveness when compared to the traditional shell-and-tube heat exchanger, or the parallel-plates heat exchanger. Another benefit with this configuration is that it is easy to construct with very thin materials and the mechanical strength is rather high with even very thin foils.

Forced convection in a triangular duct is complicated and dependent on several parameters including its apex angle, hydraulic diameter, axial length, and the flow condition. A comprehensive review of the theoretical and experimental studies on fully developed forced convection and heat transfer in a triangular duct up to the 1970s had been conducted by Shah and London [1], and documented in several well known references [1–3].

Plate-fin triangular ducts may have different heat transfer behaviors, if the fins have a limited conductivity. Baliga and Azrak [4] carried out a numerical investigation of the forced convection and heat transfer in triangular plate-fin duct. Heat conduction in the fin and forced convection in the fluid for a fully developed flow both hydrodynamically and thermally were solved as a conjugate problem. Fully developed Nusselt numbers were obtained for ducts with fin conductance from 1.0 to infinite. The results may help in exchanger design. However, as for the heat transfer in the thermally developing region, there is still no mention in the open literature. For fine ducts with relatively low speed fluid like those in air conditioning industry (4–5 m/s), heat transfer in the thermal entry length region is substantial and should be taken into account. Besides, nowadays, with the application of many new heat transfer materials like polymers [5], the fin conductance may be rather low. Regretfully there were no results yet reported

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doi:10.1016/j.ijheatmasstransfer.2006.09.013

**Nomenclature**

| | |
|---|---|
| $a$ | half duct height (m) |
| $A_c$ | cross-sectional area ($m^2$) |
| $b$ | half duct width (m) |
| $c_P$ | specific heat ($kJ\,kg^{-1}\,K^{-1}$) |
| $D_h$ | hydraulic diameter (m) |
| $f$ | friction coefficient |
| $h$ | convective heat transfer coefficient ($kW\,m^{-2}\,K^{-1}$) |
| $k$ | thermal diffusivity ($kW\,m^{-1}\,K^{-1}$) |
| $L$ | length (m) |
| $Nu$ | Nusselt number |
| $Nu_T$ | Nusselt number for thermally fully developed laminar flow with T condition |
| $P$ | pressure (Pa) |
| $P_f$ | perimeter of duct (m) |
| $Pr$ | Prandtl number |
| $Re$ | Reynolds number |
| $T$ | temperature (K) |
| $u$ | velocity ($m\,s^{-1}$) |
| $U$ | velocity coefficient |
| $x,y$ | dimensional transversal coordinates (m) |
| $z$ | axial coordinate (m) |

*Greek symbols*

| | |
|---|---|
| $\delta$ | fin thickness (m) |
| $\rho$ | density ($kg\,m^{-3}$) |
| $\theta$ | dimensionless temperature |
| $\mu$ | dynamic viscosity ($kg\,m^{-1}\,s^{-1}$) |
| $\alpha$ | half apex angle (°) |
| $\Omega$ | heat conductance parameter |

*Superscript*

| | |
|---|---|
| * | dimensionless |

*Subscripts*

| | |
|---|---|
| b | bulk |
| f | fin |
| i | inlet |
| m | mean |
| w | wall |

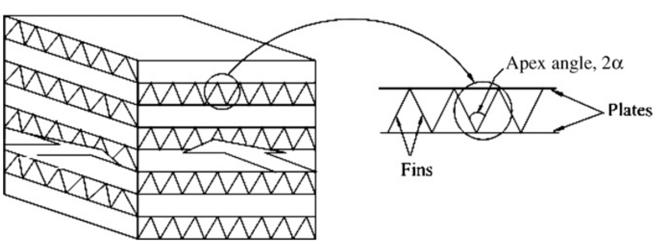


Fig. 1. Schematic of a cross-flow plate-fin heat exchanger.

for lower fin conductance less than 1.0. In a summary, there is no sufficient information on the proper design of a compact heat exchanger with new materials of lower conductance. This will be the objective of this research.

## 2. Mathematical model

The problem considered here is that of a duct shown in Fig. 2. The geometries of the triangular duct are also depicted in the figure: apex angle, 2α; height 2$a$; width 2$b$; duct length, $L$. Also plotted are two coordinate systems, $x$, $y$, $z$ for fluid and $x_1$, $y_1$, $z$ for two fins, respectively. The flow in the duct is considered to be laminar and hydrodynamically fully developed, but thermally developing in the entrance region of the duct. The fluid is Newtonian with constant thermal properties. Additionally, a uniform wall temperature boundary condition is considered. Governing dimensionless equations for the fluid are given by [6]

$$\frac{\partial^2 u^*}{\partial x^{*2}} + \left(\frac{b}{a}\right)^2 \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{4b^2}{D_h^2} = 0 \tag{1}$$

$$U\frac{\partial \theta}{\partial z^*} = \frac{\partial^2 \theta}{\partial x^{*2}} + \left(\frac{b}{a}\right)^2 \frac{\partial^2 \theta}{\partial y^{*2}} \tag{2}$$

with a dimensionless velocity

$$u^* = -\frac{\mu u}{(dP/dz)D_h^2} \tag{3}$$

and a dimensionless temperature

$$\theta = \frac{T - T_w}{T_i - T_w} \tag{4}$$

where in the equations, $T_i$ is the inlet temperature of the fluid, and $T_w$ is the wall temperature.

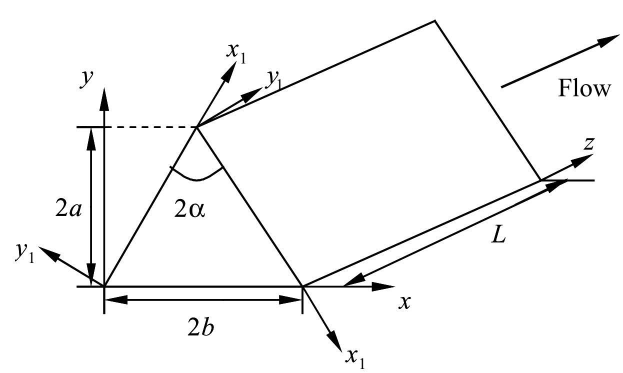


Fig. 2. Geometry of a plate-fin triangular duct.

Dimensionless coordinates are defined by

$$x^* = \frac{x}{2b} \tag{5}$$

$$y^* = \frac{y}{2a} \tag{6}$$

$$z^* = \frac{z}{D_h RePr} \tag{7}$$

where hydraulic diameter

$$D_h = \frac{4A_c}{P_f} \tag{8}$$

where $A_c$ is the cross-section area of the duct (m$^2$), $P_f$ is the perimeter of the duct (m). $Re$ is Reynolds number and $Pr$ is Prandtl number.

In Eq. (2), velocity coefficient $U$ is defined by

$$U = \frac{u^*}{u_m^*}\frac{4b^2}{D_h^2} \tag{9}$$

where $u_m^*$ is the average dimensionless velocity on a cross-section, and it is calculated by

$$u_m^* = \frac{\int\int u^* dA}{A_c} \tag{10}$$

The characteristics of fluid flow in the duct can be represented by the product of the friction coefficient and the Reynolds number as

$$(fRe) = \left(-\frac{D_h \frac{dP}{dz}}{2\rho u_m^2}\right)\left(\frac{\rho u_m D_h}{\mu}\right) = \frac{1}{2u_m^*} \tag{11}$$

Dimensionless bulk temperature

$$\theta_b = \frac{\int\int u^* \theta dA}{\int\int u^* dA} \tag{12}$$

Nusselt number

$$Nu = \frac{hD_h}{k} \tag{13}$$

where $h$ is convective heat transfer coefficient (kW m$^{-2}$ K$^{-1}$) between fluid and wall.

An energy balance in a control volume in the duct [6] will give the equation for estimation of the local Nusselt number as

$$Nu_L = -\frac{1}{4\theta_b}\frac{d\theta_b}{dz^*} \tag{14}$$

and the mean Nusselt number from $z^* = 0$ to $z^*$ by

$$Nu_m = -\frac{1}{4z^*}\ln\theta_b \tag{15}$$

The coordinate system for two fins, $x_1$, $y_1$, is shown in Fig. 2. Axis $x_1$ is along the fin surface and $y_1$ is normal to the fin surface. At any location along the fin, there is a balance between the net conduction along the fin and the heat transfer from the surface of the fin to the fluid. The heat flux at the upper surface and the lower surface are skew-symmetric [4]. Heat transfer in fin is governed by the following one-dimensional model

$$\Omega\cos\alpha\frac{d^2\theta_f}{dx_1^2} = -\left(\frac{\partial\theta}{\partial y_1^*}\right)_{x_1^*} - \left(\frac{\partial\theta}{\partial y_1^*}\right)_{1-x_1^*} \tag{16}$$

where $\Omega$ is a dimensionless parameter named fin conductance parameter. It is defined by

$$\Omega = \frac{k_f\delta}{k(2a)} \tag{17}$$

where dimensionless fin temperature

$$\theta_f = \frac{T_f - T_W}{T_i - T_W} \tag{18}$$

$$x_1^* = \frac{x_1}{L_f} \tag{19}$$

$$y_1^* = \frac{y_1}{L_f} \tag{20}$$

The boundary conditions for fluid

$$u^* = 0, \quad \text{on the 3 walls of the duct} \tag{21}$$

$$\theta = 0, \quad \text{at } y^* = 0 \tag{22}$$

Inlet condition

$$\theta = 1, \quad \text{at } z^* = 0 \tag{23}$$

The boundary conditions for fins

$$\theta_f = 0, \quad \text{at } x_1^* = 0, 1 \tag{24}$$

Fin-fluid coupling:

$$\theta_f = \theta, \quad \text{at fin–fluid interface} \tag{25}$$

## 3. Results and discussion

A boundary-fitted coordinate system is used to transform the triangular duct cross-section to a square one.

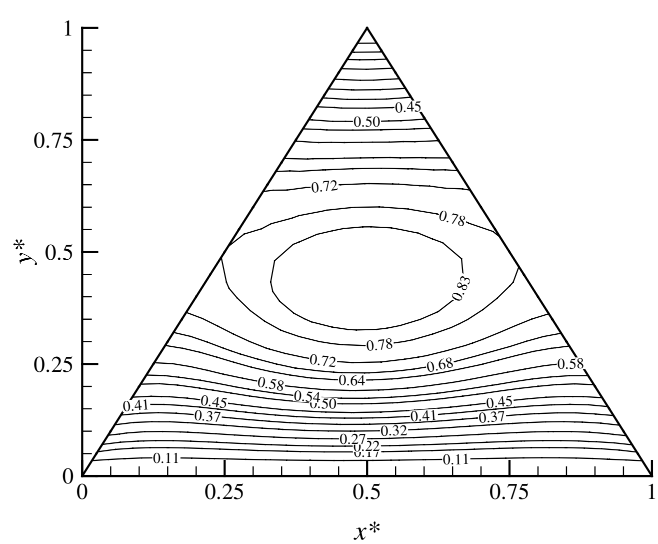


Fig. 3. Dimensionless temperature contours on duct cross-section at $z^* = 0.1$ for $\alpha = 30°$ and $\Omega = 0.1$.

Table 1
Fully developed Nusselt numbers for plate-fin triangular ducts

| α | Ω | $Nu_T$ | | α | Ω | $Nu_T$ | |
|---|---|---|---|---|---|---|---|
| | | This study | Ref. [4] | | | This study | Ref. [4] |
| 15° | ∞ | 2.391 | 2.284 | 30° | ∞ | 2.596 | 2.500 |
| | 25 | 2.287 | 2.210 | | 25 | 2.516 | 2.424 |
| | 10 | 2.196 | 2.105 | | 10 | 2.449 | 2.322 |
| | 5 | 2.062 | 1.944 | | 5 | 2.305 | 2.174 |
| | 2 | 1.599 | 1.575 | | 2 | 1.937 | 1.862 |
| | 1 | 1.288 | 1.218 | | 1 | 1.604 | 1.574 |
| | 0.5 | 1.024 | – | | 0.5 | 1.276 | – |
| | 0.1 | 0.725 | – | | 0.1 | 0.842 | – |
| | 0 | 0.651 | – | | 0 | 0.655 | – |
| 45° | ∞ | 2.451 | 2.359 | 60° | ∞ | 2.262 | 2.301 |
| | 25 | 2.393 | 2.299 | | 25 | 2.105 | 1.987 |
| | 10 | 2.299 | 2.220 | | 10 | 2.025 | 1.931 |
| | 5 | 2.182 | 2.110 | | 5 | 1.929 | 1.861 |
| | 2 | 2.021 | 1.890 | | 2 | 1.796 | 1.738 |
| | 1 | 1.744 | 1.701 | | 1 | 1.618 | 1.647 |
| | 0.5 | 1.444 | – | | 0.5 | 1.413 | – |
| | 0.1 | 0.978 | – | | 0.1 | 1.067 | – |
| | 0 | 0.659 | – | | 0 | 0.595 | – |

To assure the accuracy of the results presented, a grid independence test was performed for the duct to determine the effects of the grid size. It indicates that $21 \times 21$ grids on duct cross-section and $\Delta z^* = 0.001$ axially are adequate (less than 0.1% difference compared with $31 \times 31$ grids and $\Delta z^* = 0.0005$).

Fig. 3 shows the dimensionless temperature profiles in the fluid for $\alpha = 30°$ and $\Omega = 0.1$ at $z^* = 0.1$. As seen, At lower fin conductance parameters, temperature contours show in majority a pattern of straight lines parallel to the plate, which is similar to temperature profiles in parallel-plates ducts. In the center, the contours are circles in shape.

After the solution of temperature fields in fluid, the dimensionless bulk temperature, the local Nusselt numbers, and the mean Nusselt numbers can be calculated. After the thermal entry length, the local Nusselt number will come to a stable value $Nu_T$. This fully developed value varies with apex angles and fin conductance parameters. Table 1 lists the calculated $Nu_T$ for various half apex angles and fin conductance parameters. Also listed are some values that can be found in references to make a comparison. As can be seen, they are in agreement generally.

In designing heat exchangers, the mean Nusselt numbers in the duct, which takes into account of the thermal entry length, is more important. Following equation is regressed from the calculated values. It can be used to calculate the mean Nusslet numbers in the duct

$$Nu_m = c_1 \left( \frac{z}{D_h Re Pr} \right)^{c_2} \tag{26}$$

where $Nu_m$ is the mean Nusselt value at axial length z. The values for coefficients $c_1$ and $c_2$ are listed in Table 2. The mean Nusselt values calculated by the correlation are within 1% deviation from the numerical values.

Table 2
Constants in correlation [26]

| α | Ω | $c_1$ | $c_2$ | α | Ω | $c_1$ | $c_2$ |
|---|---|---|---|---|---|---|---|
| 15° | ∞ | 1.4628 | −0.308 | 30° | ∞ | 1.5265 | −0.3086 |
| | 25 | 1.482 | −0.2883 | | 25 | 1.5771 | −0.2895 |
| | 10 | 1.4171 | −0.2728 | | 10 | 1.6392 | −0.2616 |
| | 5 | 1.2564 | −0.27 | | 5 | 1.6796 | −0.2529 |
| | 2 | 0.9867 | −0.2738 | | 2 | 1.2812 | −0.2466 |
| | 1 | 0.7693 | −0.2901 | | 1 | 1.0351 | −0.2542 |
| | 0.5 | 0.6031 | −0.3104 | | 0.5 | 0.7301 | −0.302 |
| | 0.1 | 0.4331 | −0.3393 | | 0.1 | 0.4915 | −0.3286 |
| | 0 | 0.3954 | −0.3501 | | 0 | 0.4131 | −0.308 |
| 45° | ∞ | 1.5267 | −0.3108 | 60° | ∞ | 1.2618 | −0.3467 |
| | 25 | 1.5585 | −0.2972 | | 25 | 1.2957 | −0.3313 |
| | 10 | 1.5525 | −0.286 | | 10 | 1.2881 | −0.3212 |
| | 5 | 1.4705 | −0.28 | | 5 | 1.2253 | −0.3157 |
| | 2 | 1.2908 | −0.2688 | | 2 | 1.1056 | −0.3045 |
| | 1 | 1.0854 | −0.2714 | | 1 | 0.982 | −0.297 |
| | 0.5 | 0.8624 | −0.286 | | 0.5 | 0.8298 | −0.3007 |
| | 0.1 | 0.5789 | −0.31 | | 0.1 | 0.6187 | −0.3076 |
| | 0 | 0.4207 | −0.3438 | | 0 | 0.3928 | −0.3538 |

## 4. Conclusions

Current investigation found that, besides apex angles, the fin conductance parameter plays a leading role in heat transfer phenomenon in the plate-fin ducts. When fin conductance parameters increase from 0 to infinitely large, the duct behaves from like a duct with one well-conductive wall and two adiabatic walls to a duct with three well-conductive walls. The consequent fully developed Nusselt numbers increase accordingly. For ducts with certain limited conductance parameters, the Nusselt numbers lie between these two limiting values. This may give some new information to the heat transfer problem of plate-fin passages.

## Acknowledgements

This project is jointly supported by National Natural Science Foundation of China (50306005), Fok Ying Tung Education Foundation (101057), and Natural Science Foundation of Guangdong Province (05006557).

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